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Conjunction, Succession, Determination and Causation

MARIO BUNGE
Department of Philosophy, McGill University, Montreal

Abstract

The principles of regular (invariable or stochastic) conjunction, of retarded action, determination, and causality, are stated exactly and analyzed. To this end, the concepts of system, property, state, and event, as well as those of conjunction (of events and of properties) are first elucidated. Four types of determination are distinguished and analyzed and it is shown that only one of them qualifies as a causal nexus, the others being deterministic (in the large sense) but noncausal. The overall conclusion is that, while the principles of regular conjunction, retarded action and determination are both distinct and universal, the causal principle is just a restricted version of the principle of determinacy.

Scientific research proceeds on a number of metaphysical hypotheses, such as that nothing stands isolated, and that the present unfolds itself lawfully into the future. These hypotheses have been formulated and examined many times in the course of the last two and a half millenia but they are in need of further clarification. In particular, we should know more accurately what it means to say that two events or two properties are conjoined, that the present determines the future, and that one event determines or in particular causes another event. The aim of this paper is to attempt a clarification of these ideas along a line different from, though consistent with, some previous work on the problem (Bunge, 1959, 1961, 1962, 1963). It is hoped that the upshot will be a neater characterization of the ontological categories of conjunction, succession, determination, and causation.

1. *Preliminaries: System, Property, Event*

Since we shall deal with certain relations among properties of systems and among events, it will be convenient to start by elucidating these terms.

By a *system* we shall understand a concrete object, whether physical, biological, or cultural, acting as a unit in some respect. A field, an organism and a community are systems. For the sake of simplicity we shall restrict our attention to nonquantal systems. To indicate that an individual σ is a system of the kind Σ we shall write: $\sigma \in \Sigma$.

We assume that every system has a finite number of properties, known or unknown. We shall count relations and interactions among the properties and shall call $P(\Sigma) = \{P_1, P_2, \ldots, P_n\}$ the set of properties characterizing the class Σ. We further assume that every property P_i can be represented by a function or an operator F_i, and write '$F_i \triangleq P_i$', to indicate that F_i represents P_i. More precisely, we assume that every kind Σ of system is exhaustively characterized by a finite number n of real valued functions (or hermitian operators) F_i on Σ. In short, for every i between 1 and n, and every Σ, if $P_i \in P(\Sigma)$, then there exists an $F_i: \Sigma \to R$ such that $F_i \triangleq P_i$.† Furthermore, it will be assumed that every one of these functions or operators is basic in the sense that it cannot be defined in terms of other members of the set $P(\Sigma)$, even though it will be related to some of them. (Recall the distinction between an equation and a definition.) To this end, the amplitude and the phase of a complex-valued function shall count as two independent functions, and every component of a tensor shall count as one function. Example: all gaseous bodies of a given mass and a given chemical species are, macroscopically and ideally, characterized by three functions on the set Σ of all such bodies: the volume V, the pressure P, and the temperature T. All other macroproperties of an arbitrary gaseous body of the kind Σ will be represented by functions of these three basic functions.

A condition or *state* $s(\sigma)$ of a system of a given (non-quantal) kind Σ will be represented by an ordered n-tuple of values (or eigenvalues) of all the n basic functions (or operators) F_i that characterize Σ. Two states of σ are different if the corresponding n-tuples differ in at least one of the coordinates. Example: every state of a gaseous body σ of a given mass and chemical species is represented by an ordered triple of volume, pressure and temperature values of $\sigma: s(\sigma) \triangleq \langle V(\sigma), P(\sigma), T(\sigma)\rangle$. Likewise, the state of a person could in principle be represented by the values of a huge number of variables such as weight, sugar level, visual acuity, occupation and income.

The set of all accessible values of the basic functions of a system σ constitutes the *state space* $S(\sigma)$ of σ. In the case of the ideal gas, the state space is the cartesian product of the range of the three thermodynamic coordinates V, P, T. Although every state is assumed to be representable by an n-tuple of values, the converse is not true: not

† For the semantic relation $\triangleq$ of representing or modeling, as well as for the above analysis of properties, see Bunge, M. (1967). *Foundations of Physics*, pp. 20 ff and 30 ff. respectively. Springer-Verlag, New York.

every point of the cartesian space R^n represents a possible state. Therefore the state space is a subspace of the cartesian space R^n.

An *event* involving some system $\sigma \in \Sigma$, whether simple or complex, is any change in the state of σ. More precisely, an event $e(\sigma)$ involving the system σ will be represented by an ordered pair of states of σ, i.e. $e(\sigma) \triangleq \langle s(\sigma), s'(\sigma)\rangle$, where $s(\sigma)$ and $s'(\sigma)$ are different points in the state space $S(\sigma)$. In short, every event may be regarded as an oriented segment in this space. The identical transformation $s(\sigma) \to s(\sigma)$ is of course a nonevent or a null event: an enduring condition is not a happening. Two events are different only when the corresponding couples differ in at least one of the coordinates. Unless time occurs among the basic functions characterizing the kind of system, two events involving an individual of the kind Σ will be identical if they are represented by the same states couple irrespective of the times of occurrence. The set of possible events involving a system σ may be called the *event space* of σ. Since not every ordered pair of points in the state space is physically possible (certain transitions between states being 'forbidden'), the event space is a subspace of the cartesian product of the state space by itself: $E(\sigma) \subseteq S^2(\sigma)$.

Every space of events can be analyzed into a number of subspaces, each of which represents all the possible events of a kind. Thus temperature changes form a class of events; but, by virtue of the functional relations between temperature and several other variables, there are hardly any pure temperature changes. The set of events involving a system σ and characterized by changes in every member of a subset $A(\Sigma) \subseteq P(\Sigma)$ of the basic properties of Σ may be called a set of events of the kind E_A. Clearly, every individual scientific investigation handles only one such event subspaces.

We can now examine certain basic relations among properties and among events. We shall start with the latter, being simpler.

2. *Conjunction of Events*

Let us first analyze the idea that two events occur jointly, whether at the same place or time or not, and whether invariably or in a fixed percentage of cases.

Let $e(\sigma)$ and $e'(\sigma)'$ be two events involving the (not necessarily different) systems σ and σ' respectively. The statement that $e(\sigma)$ occurs can be symbolized thus: $e(\sigma) \in E(\sigma)$, or $e \in E$ for short; correspondingly, the assertion that $e'(\sigma')$ happens can be abbreviated to: $e' \in E'$. The *joint occurrence* of two events is a third event. The event consisting in the joint occurrence of the events e and e' will be designated by $e \cap e'$. The statement that such a compound event is the case may be formalized by saying that $e \cap e'$ is a point in the (direct product) space $E \otimes E'$, i.e., that $e \cap e' \in E \otimes E'$. (Caution: this product space does not constitute a lattice, except on the Stoic assumption that everything hangs together. The space of joint events, included in $E \otimes E'$, does constitute a lattice.)

Two events may be said to be *invariably conjoined* if, whenever one of them happens, the other occurs as well. Briefly,

Df. 1:

$$J(e, e') =_{\text{df}} e \in E \vee e' \in E' \rightarrow e \cap e' \in E \otimes E'$$

By assuming that the conjunction of events is commutative, it can be seen that the relation J of invariable conjunction is an equivalence relation: every event is trivially conjoined with itself, and the relation is also symmetric and transitive. If this analysis is correct, it invalidates Hume's analysis of causation as invariable conjunction of events, for the causal relation, though transitive, is nonreflexive and antisymmetric.

Consider now two classes of events involving systems of any kind—e.g. the class of thunderbolts and the class of shudders. Two *classes* E_A and $E_{A'}$ of events will be said to be invariably conjoined only when every member of E_A has at least one match in $E_{A'}$ and conversely, so that every pair $\langle e, e' \rangle \in E_A \times E_{A'}$ is invariably conjoined:

Df. 2:

$$J(E_A, E_{A'}) =_{\text{df}} (e)(\exists\, e')[e \in E_A \;\&\; e' \in E_{A'} \rightarrow J(e, e')] \;\&\; (e')(\exists\, e)[e' \in E_{A'} \;\&\; e \in E_A \rightarrow J(e, e')]$$

This concept of invariable conjunction of classes of events allows us to analyze Hume's *principle of invariable conjunction.* This hypothesis may be construed as asserting that, for every class E_A of events, there exists some other class $E_{A'} \neq E_A$ such that E_A and $E_{A'}$ are invariably conjoined:

$$(A)\{E_A \neq \emptyset \rightarrow (\exists\, A')[A \neq A' \;\&\; E_{A'} \neq \emptyset \;\&\; J(E_A, E_{A'})] \qquad (2.1)$$

A somewhat more refined idea of regular conjunction of events is the one of stochastic (probabilistic) conjunction. Two events e and e' will be said to be *stochastically conjoined* if their joint probability is not multiplicative:

Df. 3:

$$SJ(e, e') =_{\text{df}} \Pr(e \cap e') \neq \Pr(e) . \Pr(e')$$

Two *classes* E_A and $E_{A'}$ of events will be said to be stochastically conjoined only when every pair $\langle e, e' \rangle \in E_A \times E_{A'}$ is stochastically conjoined:

Df. 4:

$$SJ(E_A, E_{A'}) =_{\text{df}} (e)(\exists\, e')[e \in E_A \;\&\; e' \in E_{A'} \rightarrow SJ(e, e')] \;\&\; (e')(\exists\, e)[e' \in E_A \;\&\; e \in E_A \rightarrow SJ(e, e')]$$

In the absence of stochastic theories (e.g. statistical mechanics and genetics), observed relative frequencies may be used to ascertain whether two events, or two classes of events, are stochastically conjoined. But in this case, i.e. when no theoretical analysis of the possible random mechanism is at hand, one must be overcautious in concluding that two events, or two classes of events, are or fail to be stochastically conjoined. Even a coarse model assuming the stochastic independence of the events concerned will be better than nothing, for it will enable us to estimate the strength of the stochastic connection as measured by the difference between the observed frequency of the joint events and the computed value $\Pr(e) . \Pr(e')$.

We now state what may be called the *principle of stochastic conjunction*: For every class E_A of events there exists another class of events $E_{A'} \neq E_A$ such that E_A and $E_{A'}$ are stochastically conjoined:

$$(A)\{E_A \neq \emptyset \rightarrow (\exists\, A')[A \neq A' \;\&\; E_{A'} \neq \emptyset \;\&\; SJ(E_A, E_{A'})]\} \qquad (2.2)$$

Clearly, this principle subsumes the principle of invariable conjunction. Indeed, the latter follows in the particular case in which $\Pr(e \cap e') = \Pr(e) = \Pr(e')$, for then the conditional probabilities of e given e', and of e' given e, equal unity.

Finally, we shall say that two classes of events are *regularly conjoined* only when they are either invariably or stochastically conjoined. This enables us to state the general *principle of regular conjunction*: Given any kind of events, there exists another class of events, different from the former, such that the two classes are regularly conjoined.

So far, nothing has been said either about the time relations among events or about the way they are connected. Were it not a regular (invariable or stochastic) conjunction, one might take it for accidental. But of course accidental conjunctions are irregular: they do not even have a constant probability although they may exhibit a high frequency in the short run. Anyhow, theoretical science has little use for whole events, so we had better move on to the conjunction of properties.

3. *Conjunction of Properties*

Let Σ be a class of systems every one of which is treated as a unit even though it may be highly complex. Next let F_1 and F_2 be two functions on Σ, representing respectively the properties P_1 and P_2 of any given individual $\sigma \in \Sigma$, and assume as before that those functions are real valued. In short,

$$F_1 : \Sigma \to X, \qquad F_2 : \Sigma \to Y, \qquad \text{with } F_i \triangleq P_i, \quad i = 1, 2, \quad X, Y \subseteq R$$

For example, P_1 could be the bulk and P_2 the thermal agitation of a body. Correspondingly, $x \in X$ would be a volume value and $y \in Y$ a temperature value of that body.

We shall say that the properties P_1 and P_2 are concomitant, or *simply conjoined*, only when the values x of F_1 and y of F_2 for a given σ are functionally related to each other; i.e., P_1 and P_2 are simply conjoined if and only if there exists a third function G such that, for any fixed $\sigma \in \Sigma$, $y = G(x)$. Briefly,

Df. 5:

If

$$F_1 \triangleq P_1 \,\&\, F_2 \triangleq P_2 \,\&\, F_1 : \Sigma \to X \,\&\, F_2 : \Sigma \to Y \,\&\, X, Y \subseteq R$$

then:

$$C(P_1, P_2) =_{\text{df}} (\exists\, G)(G : X \to Y)$$

Example: the mass and the energy of a body are simply conjoined. By virtue of this functional relationship, the determination (in the epistemological sense) of the one enables us to determine (compute) the other. That is, given (= known or hypothesized) a value x of F_1, the function G enables us to compute the corresponding value $y = G(x)$ of F_2. It is therefore usually said that P_1 *determines* P_2. But this expression is misleading, for a functional dependence of F_2 on F_1 is not sufficient to conclude that P_1 is ontologically prior to P_2. So much so, that in most cases the function G has an inverse at least in some domain, so that $x = G^{-1}(y)$. In such cases P_1 is as much determined by P_2 as conversely. This determination is therefore a purely epistemic one, in the sense that it consists in an inference of a piece of information from another: it need have no ontological partner other than conjunction.

Things change when the conjoined properties belong to different levels of organization, for example the atomic and the molecular ones.† In most cases, if P_1 belongs to a level of organization lower than P_2, then: if P_1 and P_2 are conjoined, then P_1 *determines* P_2 but not conversely. Thus, genetic characteristics determine most phenotypical characters. But in psychology and sociology we find also many cases in which the higher determines the lower—as when a team of engineers, technicians and workers design and build a machine. In any case, if two properties belonging to different levels of organization are regularly conjoined, then one of them may determine the other in the sense that the determinant could exist without the determined but not conversely.

In most cases more than two properties are conjoined, whence the corresponding functional relations will involve more than two variables. If these functional relations (the G's of Df. 5) belong to a theoretical system (rather than being stray) and are well corroborated, they will deserve being called law statements.‡ Otherwise they will be hypotheses lacking theoretical and/or empirical support.

Two or more properties may be conjoined in yet another fashion, namely stochastically. This will be the case when the occurrence of a value $x \in X$ of F_1 determines a definite probability that the value $y \in Y$ of F_2 lies on a certain interval $[y_1, y_2]$ rather than determining uniquely the value y itself. We shall say that the properties represented by F_1 and F_2 are *stochastically conjoined* only when the values of F_1 are functionally related to the probabilities of the values of F_2, i.e. if there exists a third function G such that $\Pr(y \in [y_1, y_2]) = G(x)$, where the function Pr satisfies the postulates of the probability calculus. Briefly,

Df. 6:

If

$$F_1 \triangleq P_1 \; \& \; F_2 \triangleq P_2 \; \& \; F_1 : \Sigma \to X \; \& \; F_2 : \Sigma \to Y \; \& \; X, Y \subseteq R$$

then:

$$SC(P_1, P_2) =_{\text{df}} (\exists\, G)[G : X \to \Pr(Y)]$$

In general, either for simple conjunction or for stochastic conjunction, G will be a rather complicated function of several variables in addition to the system variable. In the simplest case, the values of

† Cf. Bunge, M. (1963). *The Myth of Simplicity*, Chapter 3. Prentice-Hall, Engelwood Cliffs, N.J. And On the Connections Among Levels. *Proceedings XIIth International Congress of Philosophy*, **VI**, 63. Sansoni, Florence (1960).

‡ For a justification of this definition of the concept of law statement, see Bunge, M. (1967). *Scientific Research*, I, Chapter 6. Springer-Verlag, Berlin–Heidelberg–New York.

F_1 and F_2 will depend not only on the system concerned but also on some parameter t, often interpreted as the time. In such a case the conjunction of properties will take either of these forms:

$$\left.\begin{array}{l} y(t) \\ \Pr(y(t) \in [y_1, y_2]) \end{array}\right\} = \mathop{G}_{\tau=t_0}^{\tau=t}[x(\tau)] \tag{3.1}$$

In particular, the functional dependence may take the integral form

$$\mathop{G}_{\tau=t_0}^{\tau=t}[x(\tau)] = \int_{t_0}^{t} d\tau . T(\tau, t)\, H[x(\tau)] \tag{3.2}$$

The set $\{x(\tau)|t_0 \leqslant \tau \leqslant t\}$ is of course the history of the system as regards its property P_1.

Thus far, the functions involved in simple and in stochastic conjunction represent *correlated properties* of a system of some kind. There has been no question of determination of one property by another (except in an epistemological sense) but only of the hanging together (invariably or stochastically) of different traits of a given system. Clearly, no causal relation is possible among properties of a single system.

The hypothesis that any given trait of a system is concomitant with at least another trait of it, is a metaphysical principle underlying scientific research. Let us state it more precisely. The *principle of the regular conjunction of properties* states that every function ('variable') characterizing any system is conjoined with at least one other function of the same system, either invariably (simple conjunction) or in a fixed percentage of cases (stochastically). In obvious symbols,

$$\begin{aligned}(\sigma)(i)\{\sigma \in \Sigma \;\&\; 1 \leqslant i \leqslant n \;\&\; F_i \triangleq P_i \rightarrow \\ (\exists j)[1 \leqslant j \leqslant n \;\&\; j \neq i \;\&\; F_j \triangleq P_j \;\&\; (C(P_i, P_j) \vee SC(P_i, P_j))]\}\end{aligned} \tag{3.3}$$

If we did not trust this metaphysical principle we would hardly care to look for relations among properties; and if this search were fruitless we would not believe the principle. This remark may be expanded into this meta-metaphysical principle: A system of metaphysics is adequate to the extent to which its principles (hypotheses) promote the search for truth by the method of science.†

† For some of the philosophical principles underlying scientific research, see Bunge, M. (1967). *Scientific Research*, I, Chapter 5, and II, Chapter 15. Springer-Verlag, Berlin–Heidelberg–New York. And *Foundations of Physics*, pp. 83–88. Springer-Verlag, New York (1967).

4. *Determination of the Present by the Past*

Consider the formulas (3.1) and (3.2) of the previous section. If the parameter t is interpreted as the (local) time and the condition $t_0 \leqslant t$ is added, these formulas say that the value of F_2 at time t—or the probability that at time t that values lies on a given bracket—is the outcome of the P_1-history of the system between t_0 and t. (Instantaneous action is obtained by choosing $H[x(\tau)] = \delta(\tau - t)$ in the second formula.) Whenever the dependence of P_2 on P_1 is nonanticipatory, or retarded, as in this case, one says that the *principle of antecedence*, or *retarded action*, is satisfied.

Formula (3.1) supplies a general formulation of this principle provided t be interpreted as the local time:

$$\left.\begin{matrix} y(t) \\ \Pr(y(t) \in [y_1, y_2]) \end{matrix}\right\} = \mathop{G}_{\tau=-\infty}^{\tau=t}[x(\tau)] \tag{4.1}$$

(In the case of an organism, the value of G will be zero between $-\infty$ and the instant its birth starts.) In particular, the dependence of the present values of P_2 on the past values of P_1 may take on the forms

$$\left.\begin{matrix} y(t) \\ \Pr(y(t) \in [y_1, y_2]) \end{matrix}\right\} = \int_{-\infty}^{t} d\tau T(\tau, t)\, H[x(\tau)] \tag{4.2}$$

The generalization of this formula to spacetime is immediate but we do not need it for our analysis.

In contemporary physics, particularly in quantum field theory and in scattering matrix theory, certain specific forms of the retarded action (or antecedence) principle are called *causality* relations or conditions. Thus it is said that the fields must satisfy the causality condition or that causality implies relativistic invariance. Yet all such 'causality' conditions state is that certain functions are so conjoined (in either of the senses of the previous section) that one of them takes its values earlier than the other. Hence those assumptions, misnamed 'causality' conditions, are just examples of what we have called the retarded action principle. So much so that they apply not only to input–output relations, some of which are genuinely causal, but also to certain pairs of functions that mirror traits of a single system—hence to situations in which there are no input and output terminals. Indeed, the formulas (4.1) and (4.2) may describe the unfolding of properties, hence of states, of an isolated system as an outcome of its inner changes. From an ontological point of view it will be a question of causal determination provided the behavior of a system under the action of another system (e.g. the milieu) is concerned, and provided certain additional conditions are satisfied (see Sections 6 and 7).

All physical systems seem to satisfy the retarded action principle. (The fact that classical electrodynamics fails to satisfy it, as it talks about the preacceleration an electron would enjoy before meeting an electromagnetic wave, is probably more of a source of embarrassment for that theory than it is for the electron.) Organisms, on the other hand, appear to be free from the retarded action stricture. Thus the bird that builds its nest would appear to have its present state determined by the future values of certain inputs. However, this is illusory: what determines the present behavior of the animal is a complex of conditions resulting from a long evolutionary process partly recorded in its genetic equipment. Likewise, what determines our rejoicing in anticipating a friendly smile is our present representation of that future event. There may well be teleology—the striving towards goals—on the higher integrative levels, but not as an action of the still nonexistent future on the present. In particular, the foreknowledge claimed by parapsychology is as impossible as magic: it is inconsistent with physics and biology—which abide by the principle of antecedence—and this is enough to write it off.

When a hypothesis is so general and becomes so deeply enmeshed in the whole fabric of science, as is the case with the retarded action principle, it attains the status of a metaphysical regulative principle employed in theory construction as well as in weeding out false conjectures and pseudodata.† Being so powerful, such metaphysical principles can become dangerous. But they may be rendered harmless by requiring (a) that they exhibit their fertility, (b) that they cohere with each other, and (c) that they be kept under critical scrutiny.

5. *Determination of one Thing by Another*

Heretofore we have treated systems as wholes, analyzing properties and events that keep company. We shall now take up an analysis of multicomponent systems. By acting upon one another, the various parts of a complex system may determine each other's behavior to some extent.

Let us assume that every multicomponent system can be analyzed into pairs of mutually acting parts. Let σ and σ' be two such subsystems, either of the same kind or of different sorts.‡ In particular, σ'

† See footnote on p. 212.

‡ For a formalization of the concepts of part, physical addition and physical product, see Bunge, M. (1967). *Foundations of Physics*, pp. 108–112. Springer–Verlag, New York.

may be the total environment of the system σ on which our attention is focused. Examples: an atom immersed in a magnetic field, a machine and its environment, an organism and its milieu. Our objects of study are then the system σ under scrutiny and the ordered pairs $\langle\sigma, \sigma'\rangle$ and $\langle\sigma', \sigma\rangle$ or, more generally, the sets Σ, $\Sigma \times \Sigma'$, and $\Sigma' \times \Sigma$.†

Suppose further that three functions (or operators) F_1, F_2 and F_3 are given, every one of which mirrors a key property of σ, $\langle\sigma', \sigma\rangle$, and $\langle\sigma, \sigma'\rangle$ respectively: $F_i \triangleq P_i$, $i = 1, 2, 3$. In particular, F_1 and F_3 may represent the same property—e.g. a force, or a light intensity. In the particular case of a system and its surrounding, P_1 is often called the *input* or stimulus of σ' on σ, P_2 a *state variable* of σ, and P_3 the *output* or response of σ on σ'. If the system σ is free or nearly so (no input), either σ' does not act upon it or it is nonexistent (in which case σ' will be the null individual of the kind Σ'). Finally, let us agree to oversimplify our analysis to the unlikely case in which only triples of properties, one per system, need be considered at a time. This is of course a pretence in the interest of perspicuity.

Our functions $F_i (i = 1, 2, 3)$ are not exactly those occurring in Section 3. Indeed, two system variables, σ and σ', and the time t will now occur. Thus $x \in X$ will be the value of the input P_1 at $\langle\sigma', \sigma, t\rangle$, while $y \in Y$ will be the value of the state property P_2 at $\langle\sigma, t\rangle$, and $z \in Z$ the value of the output P_3 at $\langle\sigma, \sigma', t\rangle$. In other words, our basic functions are now

$$F_1: \Sigma' \times \Sigma \times T \to X, \qquad F_2: \Sigma \times T \to Y, \tag{5.1}$$
$$F_3: \Sigma \times \Sigma' \times T \to Z$$

with

$$F_i \triangleq P_i, \qquad (i = 1, 2, 3, \quad X, Y, Z, T \subseteq R)$$

where T is the set of durations. If P_1, P_2 and P_3 are conjoined, the inputs will be mapped into the outputs, either in a fixed way ('deterministically') or stochastically, via a fourth function G.

The cases of practical interest are those of the action of σ' on σ (dependence of P_3 on P_1 and P_2) and $\sigma - \sigma'$ interaction (interdependence of P_1, P_2 and P_3). We assume that in both cases the principle of retarded action (Section 4) is satisfied, i.e. that the output is later than or at most simultaneous with the input. With this hypothesis, the state $F_2(\sigma, t)$ of the system at an arbitrary instant t is a

† Ordered n-tuples need not be construed as sets of sets (Wiener-Kuratowski) but may be regarded as individuals, as does S. MacLane in his homogeneous set theory (lecture at McGill University, 18 March 1968).

function, or rather a functional, of all the states prior to t, as well as of the input history during the interval $[t_0, t]$:

$$y = F_2(\sigma, t) = \underset{\tau=t_0}{\overset{\tau=t}{S}}[F_2(\sigma, \tau), F_1(\sigma', \sigma, \tau), \tau] \tag{5.2}$$

Every equation of this form may be called a *state equation* of the system σ in the milieu σ'. (Example: a linear system without memory: $dy/dt = A(t).x(t) + B(t).y(t)$.) In the absence of inputs, the system will be in a given state at every instant, and its states will unfold according to its state equation. Consequently (5.2) includes the case of free systems (null environment) evolving in a spontaneous (noncausal) way.

As regards the input-output equations, we have the following possibilities.†

1(a). Simple Action:

$$F_3(\sigma, \sigma', t) = \underset{\tau=t_0}{\overset{\tau=t}{G}}[F_1(\sigma', \sigma, t), F_2(\sigma, \tau), \tau], \qquad (t \geqslant t_0) \tag{5.3}$$

1(b). Stochastic Action:

$$\Pr(F_3(\sigma, \sigma', t) \in [z_1, z_2]) = \underset{\tau=t_0}{\overset{\tau=t}{G}}[F_1(\sigma', \sigma, t), F_2(\sigma, \tau), \tau], \qquad (t \geqslant t_0) \tag{5.4}$$

2(a). Simple Interaction:

$$\left.\begin{aligned} F_3(\sigma, \sigma', t) &= \underset{\tau=t_0}{\overset{\tau=t}{G}}[F_1(\sigma', \sigma, \tau), F_2(\sigma, \tau), \tau] \\ F_1(\sigma', \sigma, t) &= \underset{\tau=t_0}{\overset{\tau=t}{H}}[F_3(\sigma, \sigma', \tau), F_2(\sigma, \tau), \tau] \end{aligned}\right\} \quad (t \geqslant t_0) \tag{5.5}$$

2(b). Stochastic Interaction:

$$\begin{aligned} \Pr(F_3(\sigma, \sigma', t) \in [z_1, z_2]) &= \underset{\tau=t_0}{\overset{\tau=t}{G}}[F_1(\sigma', \sigma, \tau), F_2(\sigma, \tau), \tau] \\ \Pr(F_1(\sigma', \sigma, t) \in [x_1, x_2]) &= \underset{\tau=t_0}{\overset{\tau=t}{H}}[F_3(\sigma, \sigma', \tau), F_2(\sigma, \tau), \tau] \end{aligned} \quad (t \geqslant t_0) \tag{5.6}$$

Notice that in all these cases the main system or transducer, far from being a passive channel, contributes actively to the transactions.

† For an axiomatic treatment of cases 1(a) and 2(a) below, see Athans, M. and Falb, P. L. (1966). *Optimal Control*, Chapter 4. McGraw-Hill, New York.

The information-theoretical model, according to which the channel adds at most noise, is just a special case of the general systems theory.

6. *Types of Determination*

In all four preceding cases the two systems involved are physically connected in some respect, this being why the properties concerned are conjoined. The converse is not true: conjunction does not imply connection, as shown by the case of two synchronous independent clocks. The dependence among properties, hence among states and consequently among events, is very different from a mere hanging together of different traits of a single system, which concerned us in Sections 3 and 4. The dependence studied in the last Section may rightly be called *determination*, for it goes beyond an 'external' relation such as 'greater than'.

What is common to all four kinds of determination discussed above is this: on the one hand the inputs are mapped into the outputs (lawful connection) and, on the other, the outputs come after the corresponding inputs (antecedence or retarded action). The peculiarities of those kinds of determination are the following.

1(a). Simple action: P_1 and P_2 *determine* P_3. Hence changes in P_1 *cause* changes in P_3.

1(b). Stochastic action: P_1 and P_2 *determine* the *probabilities* of P_3. Consequently changes in P_1 *cause* changes in the *probabilities* of P_3.

2(a). Simple interaction: P_1 and P_2 *determine* P_3, and conversely P_3 and P_2 *determine* P_1. Hence changes in P_1 *cause* changes in P_3 and *vice versa*.

2(b). Stochastic interaction: P_1 and P_2 *determine* the *probabilities* of P_3 and, conversely, P_3 and P_2 *determine* the *probabilities* of P_1. Therefore changes in the *probabilities* of P_1 *cause* changes in the *probabilities* of P_3 and vice versa.

In all four cases certain properties are determined by other properties either simply or stochastically. (Needless to say, in the latter case the probabilities in question, or rather the random variables involved, are objective physical properties not measures of our ignorance.) Correspondingly every change in the degree of some property (i.e. every event of a kind) has some effect, of the same or a different kind, as the events that trigger a change in the state of the compound system. The four input-output relations studied above fit therefore into determinism *lato sensu* or neodeterminism.† Indeed, thus weakened determinism asserts only:

(i) that every thing and every event emerge from preexisting conditions in some system (*genetic* hypothesis or non-magic postulate), and
(ii) that every property is lawfully conjoined to some other properties, either simply or stochastically (*lawfulness* hypothesis or postulate of regular conjunction).

The term *indeterminist* to characterize systems or theories containing random variables was justified before the birth of stochastic physics: it has now become a misnomer for it suggests the denial of either or both the above postulates. Chance is increasingly being recognized as an objective mode of behavior, even if all variables turned out to be basically nonrandom ('hidden' variables). And if this behavior satisfies both the principle of antecedence [contained in the genetic hypothesis (i)] and stochastic laws, then it is deterministic in the large sense. Only that which comes out of nothing or gets annihilated, and at the same time satisfies no law, deserves to be called *indeterminate* (or *indeterministic*), for it is determined by nothing, not even by its own past history. If anything like this were to exist, it would be impregnable to scientific research, which is a methodical search for pattern. Since science refuses to acknowledge the existence of objects totally and forever adamant to research, it thereby rejects indeterminism. Science is now as deterministic as it was in the days of Laplace or even Bernard, but not exactly in the same sense, for it has discovered types of determination that were unknown at that time. Whence the need for rejuvenating determinism. If metaphysics is to keep the pace of science and if it is to be cooperative rather than obstructive, every major metaphysical principle must be overhauled from time to time.

7. *Causality*

In all four cases envisaged in the last section, initial changes of state determine changes in the final state of a compound system (e.g. a transducer coupled to its environment). Such events are not merely associated or conjoined in the sense of Section 2. In the present situation every initial change *produces* (engenders, brings about) one

† For an analysis of general determinism, see Bunge, M. (1959). *Causality*. Harvard University Press, Cambridge, Mass. Meridan Books, Cleveland and New York (1963).

or more final changes. Therefore these events (changes of state) deserve being called *causes* or *effects*. (On the other hand a system and a property do not qualify as causes: only changes can have causal efficacy.)

However the *relation* between a set of causes and a set of effects need not be causal: there are noncausal relations among causes and effects, as exemplified by 1(b), 2(a) and 2(b) in Sections 5 and 6. Only *simple action* (case 1(a)) can qualify for causality: the remaining types of determination are far more complex than causal determination. This is not a logical or an empirical question but one of terminology: the philosophical tradition, which was not born yesterday, does not happen to call *causation* an interaction or a stochastic relation. Never mind if some physicists claim that every differential equation expresses a causal nexus, or if on second thought they stipulate that only canonical equations qualify, or if they mistake retarded action for causation: since there is no tradition and no system behind such caprices, we need not abide by them.

For a relation between two properties to deserve being called *causal*, it must satisfy the following requirements:

C1: The relation must involve at least *two different systems*, the determiner and the determined ones. The relation between two properties of one and the same system is not causal even if they belong to different levels of organization of the system.

C2: The properties and events concerned must be *regularly conjoined*, i.e. they must be associated either simply or stochastically (Sections 2 and 3). That is, causes and effects must be lawfully related.

C3: The actions must be *retarded*; i.e., there is to be nonnegative delay between the cause and the effect.

C4: The output shall effect negligible changes in the determiner: the *feedback must be unimportant.*

C5: The properties of the determined system shall have *negligible spontaneous fluctuations*: there must not be spontaneous effects.

Aside from these conditions, causal relations can be of either of these types:

(a) *Plurality of causes*: a number of different causes may disjunctively (each by itself) produce a given effect;
(b) *Plurality of effects*: any given cause may alternatively produce a number of effects;
(c) *Simple causation*: one-to-one reciprocal correspondence between causes and effects.

Every one of these types of causation is housed in its own version of the so-called *principle of causality*:

(a) 'Every event has at least one cause'.
(b) 'Every event has at least one effect'.
(c) 'Every event has exactly one cause and every cause has exactly one effect.'

Now, the 'may' occurring in the statements of multiple causation is inconsistent with the necessity that the word 'causation' usually evokes: where there is leeway there would seem to be a nexus far less strict than the causal nexus. Therefore we may wish to restrict the latter to simple (one-to-one reciprocal) causation. In other words, in its *strict* sense causation is the relation characterized by the properties C1 through C5 above, plus:

C6: The causes and the effects must be related in a one-to-one reciprocal fashion. (The transducer functions G that map inputs into outputs must be one to one and onto.)

According to our analysis, the causal nexus is just one among four different types of determination. In actual fact few if any real systems are found to be coupled in a strictly causal way, which is not surprising, for the idea of causality is older than modern science. (The idea of conjunction seems to be even earlier, both psychogenetically and historically.) Every real system is acted on by random stimuli which are partly absorbed by it rather than being faithfully mapped into its output. Many systems exhibit a spontaneous fluctuation in some properties, even in the absence of external stimulation. And all systems have some 'life' of their own, in the sense that they can change of their own accord, i.e. without an external cause. The idea that nothing changes unless acted on by external agents is an Aristotelian tenet.

For example, the sensory response $R(t)$ of an organism in a certain respect and at time t is possibly constituted by (a) the value of a functional of the stimuli S conjoined with R that have acted on the organism during its history, (b) the state variables lumped in the symbol 'y', and (c) a spontaneous response term $U(x,t)$, where 'x' summarizes certain inner variables (e.g., at the neural level):

$$R(t) = \underset{\tau=t_0}{\overset{\tau=t}{G}}[S(\tau), y(\tau), \tau] + \underset{x=a}{\overset{x=b}{U}}(x,t)$$

In this case it is clear that the *causal range* of this law is the subset of ordered pairs $\langle$input, output$\rangle$ for which $|U| \ll R$. But in other cases there will be a response without a stimulus, and in still other cases the system concerned will be equipped with a shock-absorbing mechanism, so that its transducer function G will map every input, within bounds, into zero. In all such cases the laws concerned will have a narrow or even a vanishing causal range: they will be altogether noncausal. Which shows that the popular identities 'Lawfulness = Causality' and 'Causality = Determinism' are mistaken.

8. *Closing Remarks*

We have analyzed two main kinds of physical relation: conjunction and determination. Conjunction or togetherness can be of events or of properties. Determination can be of the present by the past, or of one level by another, or of one thing by another. In every case objective relations are supposed to be at stake.

Causation has turned out to be a very special kind of determination and, moreover, a nexus that is far from being universal. Therefore any failures of causality do not count as refuters of determinism *lato sensu*, i.e. lawfulness conjoined with nonmagic. Taken in this sense, determinism is indispensable to scientific research and it is confirmed by every success of the latter. Moreover, unlike scientific hypotheses the metaphysical hypothesis of determinism *lato sensu* is irrefutable for, if anything should look lawless or seem to come out of the blue or go into the blue, we would ask for, and be granted, all the time necessary to refute such an impression.

In brief, the principles of regular conjunction or lawfulness (whether simple or stochastic), and the genetic principle (which subsumes the principle of antecedence) are neither laboratory results nor metaphysical illusions: they are no less than presuppositions of scientific research. They amount to the hypothesis that, no matter how chaotic an arbitrarily chosen set of events may be, the world as a whole is basically ordered and self-regenerating. As to the causal principle, even though it constitutes a very special form of the principle of determinism, it is part of the philosophic engine of scientific research. Every time we proclaim its universal extension we err. But every time we adopt it as a working hypothesis and as holding to a first approximation, we find something—often a noncausal relation satisfying a richer type of determination.

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